14.6 Damped Oscillations

14. For a damped oscillation, is the time constant $\tau$ greater than or less than the time in which the oscillation amplitude decays to half of its initial value? Explain.

\[
\begin{align*}
X_{\text{max}}(t) &= X_{\text{max}}(T_\text{o}) e^{-\frac{t}{\tau}} \\
\frac{1}{2} X_{\text{max}}(T_\text{o}) &= X_{\text{max}}(t) e^{-\frac{t}{\tau}} \\
\ln(\frac{1}{2}) &= -\frac{1}{\tau} \\
\ln(\frac{1}{2}) &= -\frac{t}{\tau} \\
\frac{1}{\tau} &= e^{-\frac{t}{\tau}} \\
\gamma &= \frac{1}{\ln(2)} \tau_{1/2}
\end{align*}
\]

\[\gamma > \tau_{1/2}\]

15. The amplitude of a damped oscillation decays to one-half of its initial value in 4.0 s. How much additional time will it take until the amplitude is one-quarter of its initial value? Explain.

\[\gamma = \frac{4\tau}{\ln(4)}
\]

\[\ln(\frac{1}{2}) = -\frac{t}{\tau}
\]

\[\ln(\frac{1}{2}) = -\frac{t}{\tau}
\]

\[\ln(\frac{1}{2}) = -\frac{t}{\tau}
\]

\[\frac{1}{\tau} = \frac{1}{\ln(2)} \tau_{1/2}
\]

\[\gamma = \frac{1}{2} \frac{1}{\ln(2)} \tau_{1/2}
\]

\[\gamma = 2 \tau_{1/2}
\]

16. If the time constant $\tau$ of an oscillator is decreased, do the oscillations die away more quickly or less quickly? Explain. MORE QUICKLY

\[X_{\text{max}}(t) \propto e^{-\frac{t}{\tau}}
\]

\[\text{as } \tau \downarrow e^{\frac{1}{\gamma}} \uparrow \text{ so } e^{\frac{t}{\gamma}} \text{ approaches zero quicker.}
\]

17. The figure below shows the decreasing amplitude of a damped oscillator. (The oscillations are occurring rapidly and are not shown; this shows only their amplitude.) On the same axes, draw the amplitude if (a) the time constant is doubled and (b) the time constant is halved. Label your two curves “doubled” and “halved.”
15.1 The Wave Model

15.2 Traveling Waves

1. a. In your own words, define what a *transverse wave* is.

   b. Give an example of a wave that, from your own experience, you know is a transverse wave. What observations or evidence tells you this is a transverse wave?

2. a. In your own words, define what a *longitudinal wave* is.

   b. Give an example of a wave that, from your own experience, you know is a longitudinal wave. What observations or evidence tells you this is a longitudinal wave?

3. Three wave pulses travel along the same string. Rank in order, from largest to smallest, their wave speeds $v_1$, $v_2$, and $v_3$.

   Order: $v_1 < v_2 < v_3$

   Explanation:

   \[ v = \sqrt{\frac{T}{\mu}} \]

   Same rope...
   Same $\mu$...
   \[ \therefore \text{ same } v \]
4. A wave pulse travels along a string at a speed of 200 cm/s. What will be the speed if:
   **Note:** Each part below is independent and refers to changes made to the original string.
   a. The string’s tension is doubled?
      \[ v = \sqrt{\frac{F}{\mu}} \]
      \[ v \rightarrow \sqrt{2} \cdot v \]
   b. The string’s mass is quadrupled (but its length is unchanged)?
      \[ v \propto \frac{1}{\sqrt{L \cdot m}} \]
      \[ v \propto \frac{1}{\sqrt{4 \cdot m}} \]
      \[ v \rightarrow \frac{1}{2} \cdot v \]
   c. The string’s length is quadrupled (but its mass is unchanged)?
      \[ v \propto \frac{1}{\sqrt{L \cdot m}} \]
      \[ v \propto \frac{1}{\sqrt{4L \cdot m}} \]
      \[ v \rightarrow \frac{1}{2} \cdot v \]
   d. The string’s mass and length are both quadrupled?
      \[ v \propto \frac{1}{\sqrt{L \cdot m}} \]
      \[ v \propto \frac{1}{\sqrt{4L \cdot 4m}} \]
      \[ v \rightarrow \frac{1}{2} \cdot v \]

5. Sound travels through a 300 K gas at 400 m/s. What will be the sound speed if the gas temperature is increased to 600 K? Explain.
15.3 Graphical and Mathematical Descriptions of Waves

6. Each figure below shows a snapshot graph at time \( t = 0 \) s of a wave pulse on a string. The pulse on the left is traveling to the right at 100 cm/s; the pulse on the right is traveling to the left at 100 cm/s. Draw snapshot graphs of the wave pulse at the times shown next to the axes.

a. 

\[ y (\text{mm}) \]

\[ t = -0.01 \text{ s} \]

\[ x (\text{cm}) \]

\[ t = 0.00 \text{ s} \]

\[ 100 \text{ cm/s} \]

\[ t = 0.01 \text{ s} \]

\[ t = 0.02 \text{ s} \]

\[ t = 0.03 \text{ s} \]

\[ t = 0.04 \text{ s} \]

b. 

\[ y (\text{mm}) \]

\[ t = -0.01 \text{ s} \]

\[ x (\text{cm}) \]

\[ t = 0.00 \text{ s} \]

\[ 100 \text{ cm/s} \]

\[ t = 0.01 \text{ s} \]

\[ t = 0.02 \text{ s} \]

\[ t = 0.03 \text{ s} \]

\[ t = 0.04 \text{ s} \]

7. This snapshot graph is taken from Exercise 6a. On the axes below, draw the history graphs \( y(x = 2 \text{ cm}, t) \) and \( y(x = 6 \text{ cm}, t) \) showing the displacement at \( x = 2 \text{ cm} \) and \( x = 6 \text{ cm} \) as functions of time. Refer to your graphs in Exercise 6a to see what is happening at different instants of time.
8. This snapshot graph is from Exercise 6b.
   a. Draw the history graph \( y(x = 0 \text{ cm}, t) \) for this wave at the point \( x = 0 \text{ cm} \).
   b. Draw the \textit{velocity}-versus-time graph for the piece of the string at \( x = 0 \text{ cm} \). Imagine painting a dot on the string at \( x = 0 \text{ cm} \). What is the velocity of this dot as a function of time as the wave passes by?
   c. As a wave passes through a medium, is the speed of a particle in the medium the same as or different from the speed of the wave through the medium? Explain.

\[ V_{\text{wave}} \text{ is different than } V_{\text{particle}} \]
\[ V_{\text{wave}} \text{ is constant} \]
\[ V_{\text{particle}} \text{ oscillates} \]

9. Below are four snapshot graphs of wave pulses on a string. For each, draw the history graph at the specified point on the \( x \)-axis. No time scale is provided on the \( t \)-axis, so you must determine an appropriate time scale and label the \( t \)-axis appropriately.

a. \( y \) \( \quad \text{At } t = 0 \text{ s} \quad \) \( 100 \text{ cm/s} \)
\[ \begin{array}{cc}
2 & 4 \\
4 & 6 \\
\end{array} \quad x \text{ (cm)} \]
\[ \begin{array}{cc}
\text{At } x = 2 \text{ cm} \\
\text{At } x = 8 \text{ cm} \\
\end{array} \]

b. \( y \) \( \quad \text{At } t = 0 \text{ s} \quad \) \( 100 \text{ cm/s} \)
\[ \begin{array}{cc}
2 & 4 \\
4 & 6 \\
\end{array} \quad x \text{ (cm)} \]
\[ \begin{array}{cc}
\text{At } x = 2 \text{ cm} \\
\text{At } x = 8 \text{ cm} \\
\end{array} \]

d. \( y \) \( \quad \text{At } t = 0.02 \text{ s} \quad \) \( 100 \text{ cm/s} \)
\[ \begin{array}{cc}
2 & 4 \\
4 & 6 \\
\end{array} \quad x \text{ (cm)} \]
\[ \begin{array}{cc}
\text{At } x = 0 \text{ cm} \\
\text{At } x = 0 \text{ cm} \\
\end{array} \]
12. The figure shows a sinusoidal traveling wave. Draw a graph of the wave if:

a. Its amplitude is halved and its wavelength is doubled.

b. Its speed is doubled and its frequency is quadrupled.

\[
\begin{align*}
\gamma &= \frac{f}{\lambda} \\
\gamma' &= 2\gamma \\
\lambda' &= \frac{1}{4}\lambda \\
\gamma' &= 4\gamma \\
\lambda' &= \frac{1}{4}\lambda
\end{align*}
\]

13. The wave shown at time \( t = 0 \) s is traveling to the right at a speed of 25 cm/s.

a. Draw snapshot graphs of this wave at times \( t = 0.1 \) s, \( t = 0.2 \) s, \( t = 0.3 \) s, and \( t = 0.4 \) s.

b. What is the wavelength of the wave? 10 cm

c. Based on your graphs, what is the period of the wave? 0.4 s

d. What is the frequency of the wave?

\[
\begin{align*}
F &= \frac{1}{T} = \frac{1}{0.4} = 2.5 \text{ Hz}
\end{align*}
\]

e. What is the value of the product \( \lambda f \)?

\[
\lambda f = (10 \text{ cm})(2.5 \text{ Hz}) = \frac{25 \text{ cm}}{3}
\]

f. How does this value of \( \lambda f \) compare to the speed of the wave?

\[
\lambda f = v
\]
15. We can use a series of dots to represent the positions of the links in a Slinky. The top set of dots shows a Slinky in equilibrium with a 1-cm spacing between the links. A wave pulse is sent down the Slinky, traveling to the right at 10 cm/s. The second set of dots shows the Slinky at \( t = 0 \) s. The links are numbered, and you can measure the displacement \( \Delta x \) of each link from its equilibrium position.

\[
\begin{array}{ccccccccccccccc}
0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 \\
\text{Slinky in equilibrium} & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\
\text{Wave pulse at } t = 0 \text{ s} & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\
\end{array}
\]

\[\text{1 cm}\]

\[\begin{array}{cccccccccccccccc}
0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & \text{10 cm/s} \\
\end{array}\]

a. Draw a snapshot graph showing the displacement \( \Delta x \) of each link at \( t = 0 \) s. There are 13 links, so your graph should have 13 dots. Connect your dots with lines to make a continuous graph.

b. Is your graph a “picture” of the wave or a “representation” of the wave? Explain.

c. Which links are in compression? (list their numbers)

Which links are in rarefaction? (list their numbers)

16. Rank in order, from largest to smallest, the wavelengths \( \lambda_1 \) to \( \lambda_3 \) for sound waves having frequencies \( f_1 = 100 \text{ Hz} \), \( f_2 = 1000 \text{ Hz} \), and \( f_3 = 10,000 \text{ Hz} \).

Order: \( \lambda_3 \lesssim \lambda_2 \lesssim \lambda_1 \)

Explanation:

\[\sqrt[\omega/\lambda] = \frac{c}{\omega} \]

\[\omega/\lambda = c/\omega_f \]

\[\lambda \propto \frac{1}{f} \]

\[\rho \propto \frac{1}{f} \cdot \lambda \]
15.5 Energy and Intensity

15.6 Loudness of Sound

17. The figure shows the path of a light wave past a series of equally spaced transparent grids. The portion of the first two grids that would be illuminated by the light is represented by the shaded area in the first two grids.

a. Complete the figure by shading in the spaces that would be illuminated in the remaining two grids.

b. Is the energy of the light wave passing through the fourth transparent grid (D) greater than, less than, or equal to the energy passing through the first transparent grid (A)? Explain.

"SAME ENERGY BUT IT IS SPREAD OUT OVER A LARGER SURFACE AREA."

c. What is the ratio \( I_D/I_A \) of the intensity of the light at the fourth grid (D) to the intensity at the first grid (A)? Explain.

\[
I = \frac{P}{A}, \quad I = \frac{E}{AT} = \cos^2 \theta \quad A \rightarrow 16 A
\]

\[
I = \frac{I_A}{A} \quad I \rightarrow \frac{1}{16} I
\]

18. A laser beam has intensity \( I_0 \).

a. What is the intensity, in terms of \( I_0 \), if a lens focuses the laser beam to \( \frac{1}{10} \) its initial diameter?

\[
I = \frac{P}{A} \quad \omega' = \cos \theta \quad I \omega' \sim \frac{1}{\pi r^2} \quad I \sim \frac{1}{r^2} \quad \frac{1}{r^2} \quad I \sim 100 I
\]

b. What is the intensity, in terms of \( I_0 \), if a lens defocuses the laser beam to 10 times its initial diameter?

\[
I \sim \frac{1}{d^2} \quad I \sim 10 d \quad I \rightarrow \frac{1}{100} I
\]
19. Sound wave A delivers 2 J of energy in 2 s. Sound wave B delivers 10 J of energy in 5 s. Sound wave C delivers 2 mJ of energy in 1 ms. Rank in order, from largest to smallest, the sound powers $P_A$, $P_B$, and $P_C$ of these three sound waves.

Order: $P_A < P_B = P_C$

Explanation:

\[ P_A = \frac{2}{2} \text{ W} = 1 \text{ W} \]

\[ P_B = \frac{10}{5} \text{ W} = 2 \text{ W} \]

\[ P_C = \frac{2 \times 10^{-3}}{1 \times 10^{-3}} \text{ W} = 2 \text{ W} \]

20. A giant chorus of 1000 male vocalists is singing the same note. Suddenly, 999 vocalists stop, leaving one soloist. By how many decibels does the sound intensity level decrease? Explain.

\[ L'_I = 10 \text{ dB } 10 \log_{10} \left( \frac{1}{1000} \right) \]

\[ L'_I = 10 \text{ dB } k \log_{10} \left( \frac{1000}{1} \right) \]

\[ L_B = 10 \text{ dB } \log_{10} \left( \frac{1}{1000} \right) \]

\[ L_B = 10 \text{ dB } \log_{10} \left( \frac{1}{1000} \right) \]

\[ L_B = 10 \text{ dB } \log_{10} \left( \frac{1}{1000} \right) \]

\[ L_B = 10 \text{ dB } \log_{10} \left( \frac{1}{1000} \right) \]

\[ L_B = 10 \text{ dB } \log_{10} \left( \frac{1}{1000} \right) \]

\[ L_B = 10 \text{ dB } \log_{10} \left( \frac{1}{1000} \right) \]

\[ L_B = 10 \text{ dB } \log_{10} \left( \frac{1}{1000} \right) \]

\[ L_B = 10 \text{ dB } \log_{10} \left( \frac{1}{1000} \right) \]

\[ L_B = -30 \text{ dB} \]

The sound intensity level decreases by 30 dB.
\( l = 0.20 \text{ m} \)

\[ M_s = 7.0 \times 10^{-4} \text{ kg} \]

\[ \text{For string: } \dot{v} = \frac{\frac{1}{2} \pi \rho \pi d^2}{\frac{1}{4} \pi \rho \pi d^2} \]

\[ \dot{v} = \frac{4 m_s g}{\pi \rho \pi d^2} \]

\[ \dot{v} \approx 1301 \text{ m/s} \]

\[ \Delta t \approx 1.5 \times 10^{-4} \text{ sec} \]

\[ \frac{\Delta y}{\Delta t} = \gamma \dot{v} + \frac{1}{2} g \Delta t^2 \]

\[ \Delta y = \gamma \Delta t \]

\[ \Delta t = \frac{\Delta y}{\gamma} \]

\[ \text{Stick } \Delta t = 2 \times 10^{-6} \text{ m} \]
\[ F_{r} = m_s a_r \]
\[ |\vec{F}_r| + |\vec{F}_{es}| = m_s a_r \]
\[ |\vec{F}_r| + m_s g = m_s \frac{V_t^2}{r} \]
\[ |\vec{F}_r| + m_s g = m_s \omega^2 r \]
\[ |\vec{F}_r| = m_s \omega^2 r - m_s g \]
\[ |\vec{F}_r| = m_s \left( \frac{4\pi^2 F^2 \pi}{g} \right) \]
\[ V = \sqrt{\frac{|\vec{F}_r|}{\mu}} = \sqrt{\frac{m_s \left( \frac{4\pi^2 F^2 \pi}{g} \right)}{\mu}} \approx 1.94 \text{ m/s} \]
\[ \Delta t = \frac{L}{V} \approx 1.03 \times 10^{-5} \text{ sec} \]
\[ \Delta t = L \sqrt{\frac{\rho \pi d^2}{y m_s \left( 4\pi^2 F^2 \pi + g \right)}} \approx 7.49 \times 10^{-5} \text{ sec} \]