LECTURE 08: 2-D Kinematics

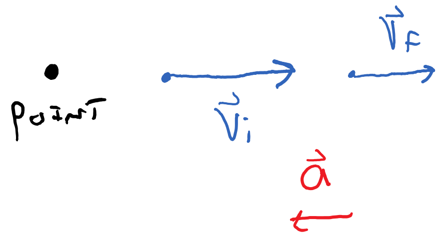
Select LEARNING OBJECTIVES:

i. Be able to identify when a multi-dimensional analysis is necessary.
ii. Demonstrate the ability to describe the motion of an object in 2-D scenarios through the use of kinematic equations with constant acceleration.
iii. Understand the vector nature of kinematic equations. Specifically how x and y components are decoupled under the constraint of no air resistance.
iv. Demonstrate the ability to quantitatively solve algebraic expressions, including quadratic equations.
v. Begin the development of problem solving strategies, which includes but is not limited to translating problems into different representations, determining knowns/unknowns, and identifying the relevant physics.
vi. Demonstrate the ability to construct kinematic equations with the knowns and unknowns which are specific to a problem.

TEXTBOOK CHAPTERS:

- Giancoli (Physics Principles with Applications 7th) :: 3-5 ; 3-6 ; 3-7
- Knight (College Physics : A strategic approach 3rd) :: 3.6 ; 3.7
- BoxSand :: Kinematics (2D Kinematics)

WARM UP: T/F? If an has an acceleration towards a point, then it must be getting closer to that point.



WARM UP: Assuming equal rates of acceleration in both cases, how much further would you travel if braking from 56 mph to rest than from 28 mph to rest.

1) 2 times further
2) 4 times further
3) 3.2 times further
4) 4.8 time further
5) 5.2 times further

$$V_{fx}^2 = V_{ix}^2 + 2a_x \Delta X$$

CASE A

$V_{ix_A} = 28_{MPH}$

$V_{fx_A} = 0$ MPH

$a_{xA} = a_{xB}$

CASE B

$V_{ixB} = 56$ MPH $= 2 V_{ixA}$

$V_{fxB} = 0$ MPH

$a_{xB} = a_{xA}$

$$0 = V_{ix}^2 + 2a_x \Delta X$$

w/ a_x = CONSTANT FOR A AND B

$$V_{ix}^2 \propto \Delta X$$

IF $V_{ix} \uparrow 2$ $\quad \Delta X \uparrow 4$

We will now begin to study the motion of an object in 2 dimensions. In this lecture we will look at the mathematical description of motion (i.e. kinematics). This 2-D kinematics section will follow most all of the same constraints that our 1-D kinematics were subject to. Just as a quick reminder, these constrains relevant to 2-D kinematics are…

We will not ask questions about what causes the motion.

We will be using the point particle model for objects.

This lecture will also only look at constant acceleration scenarios.

Also, air resistance will be ignored (constant acceleration implies this simplification).

Ok so let's jump right in by starting off defining the kinematic equations in 2-D.

2-D Kinematic Equations

1) $\vec{r}_f = \vec{r}_i + \vec{v}_i \Delta t + \frac{1}{2}\vec{a}\Delta t^2$

2) $\vec{v}_f = \vec{v}_i + \vec{a}\Delta t$

3) $v_{fx}^2 = v_{ix}^2 + 2a_x \Delta x$

$v_{fy}^2 = v_{iy}^2 + 2a_y \Delta y$

*Note about notation: The above subscripts have the following meanings…

f --> final
i --> initial
x --> x-coordinate

Look carefully at these kinematic equations above. Equation 1 and 2 are in vector form, so each equation is really two equations. You should be comfortable with this statement at this point. If not please go back and review some of the lectures that relate to vectors. Equations labeled 3 are already broken up into component form since we don't really have a nice vector notation for the operation that produces those equations (ok we actually do, but no one uses it).

When we covered vectors, we learned that we add vectors component wise. Perhaps at the time it was just an axiom that you were willing to accept because I told you so. But now we can put some context behind why that is true. The context I am talking about is 2-D motion. Grab an object that is nearby, perhaps an apple, now walk at a brisk pace with the apple in your hand. With your palm facing upwards, toss the apple vertically into the air while continuing to walk. The apple will land right back into your hand even though you kept walking forward. This little demonstration is an example of how the vertical motion (apple being thrown upwards), does not affect the horizontal motion (apple's initial horizontal component of velocity). I want to say that again; the vertical motion of an object does not affect the horizontal motion of that object. Keep in mind, we are under the constraint of no air resistance, which if included would change this statement.

SKIP THIS SECTION - DERIVATIONS (Calculus bonus)

Recall that derivatives and integrals are linear operators. Therefore there are no new derivations to cover this lecture. There would just be two components that we would write down instead of the one from the 1-D scenario.

DERIVATIVES ARE LINEAR OPERATORS....

$$\vec{v}(t) = \frac{d\vec{r}(t)}{dt} = \frac{d}{dt}\left[\langle r_x, r_y \rangle\right] = \left\langle \frac{dr_x}{dt}, \frac{dr_y}{dt} \right\rangle$$

INTEGRALS ARE LINEAR OPERATORS....

$$\vec{r}(t) = \int_{t_i}^{t_f} \vec{v}(t)\,dt = \int_{t_i}^{t_f} \left[\langle v_x(t), v_y(t) \rangle\right] dt = \left\langle \int_{t_i}^{t_f} v_x(t)\,dt, \int_{t_i}^{t_f} v_y(t)\,dt \right\rangle$$

PROBLEM SOLVING TECHNIQUES

These are the same as the 1-D problem solving techniques! Just be careful to separate components and apply the kinematic equations appropriately. In other words, you now have a total of 6 kinematic equations, two from equation labeled 1 above, two form equation labeled 2 above, and the two from equation labeled 3.

A common mistake is to split Δt into Δt_x and Δt_y. Time is a scalar. It makes no sense to talk about x and y components of time. The amount of time an object spends in the air is just a single scalar. So the Δt you see in both the x and y component forms of the kinematic equations is the same Δt.

2-D kinematics involves scenarios where there exists an acceleration in both the x and y direction. Unfortunately due to time constraints, we will only cover projectile motion which is a special case of 2-D motion where the acceleration of the object of interest is only the acceleration due to gravity in the downwards direction.

Conceptual questions for discussion

1) Assuming that air resistance is negligible for the first few seconds of projectile motion when an object is released from a plane in flight, describe the object's trajectory you would see when looking down from your plane.
2) Sphere A moves across a level track, like the one seen below in red, at a constant velocity. If sphere B moves along a parallel track, like the one seen below in green, with an initial velocity equal to that of sphere A, which sphere will reach the finish line first? Or will they reach at the same time? How do their velocities compare at the finish line?