LECTURE 13: Translating free body diagrams

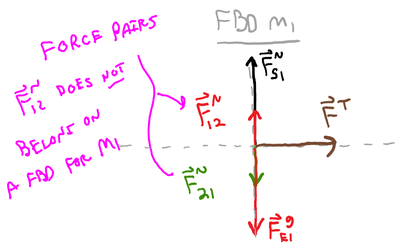
Select LEARNING OBJECTIVES:

i. Demonstrate the ability to translate a FBD into Newton's second law equations.
ii. Strengthen the ability to decompose vectors into components along the chosen coordinate system.

TEXTBOOK CHAPTERS:

- Giancoli (Physics Principles with Applications 7th) :: 4-7
- Knight (College Physics : A strategic approach 3rd) :: 5.1, 5.2, 5.3
- BoxSand :: Forces (Newton's Second Law)

WARM UP: Discuss the validity of the free body diagram below.

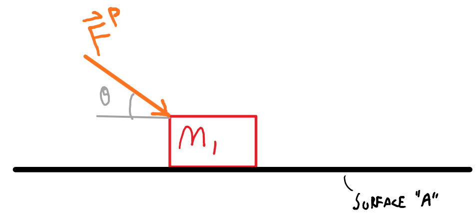


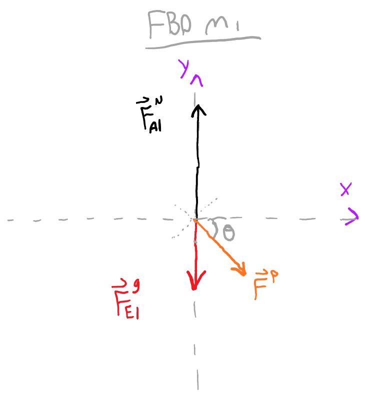
Free body diagrams are an invaluable tool. Up to this point we have only looked at constructing FBDs. We will now use Newton's second law to translate our FBDs into a system of equations from which we can determine the resulting motion of our object. It is crucial that our FBD is complete (coordinate system present, all forces included and properly labeled, and any relevant angles defined) before we begin this translation process.

The basics are as follows: Sum up all the forces acting on your system (i.e. all the forces in your FBD) and setting the result to the mass of the system times the acceleration of the center of mass. Remember, forces are vectors so our summation process will include breaking the force vectors into the components along the chosen coordinate system, then adding all the x-components together and adding all the y-components together. These summation of x and y components are then set equal to the respective system mass times the x and y components of the acceleration of the systems center of mass. Furthermore, we can then simplify the equation(s) we have just written down by putting in any definitions such as the magnitude of the force of gravity which is equal to mass times the acceleration due to gravity. We should also simplify the equations by inserting any known information such as accelerations that are equal to zero if the net force in one direction is zero.

Let's show one example and then do some practice problems.

EXAMPLE: Draw a FBD for the mass below. Attempt to scale each force relative to each other. Then translate each FBD into equations using Newton's 2nd law.





$\Sigma F_y = m a_y$

$F^N_{A1y} - F^g_{E1y} - F^P_y = m_1 \cancel{a_y}^{0}$

$|\vec F^N_{A1}| - |\vec F^g_{E1}| - |\vec F^P| \sin(\theta) = 0$

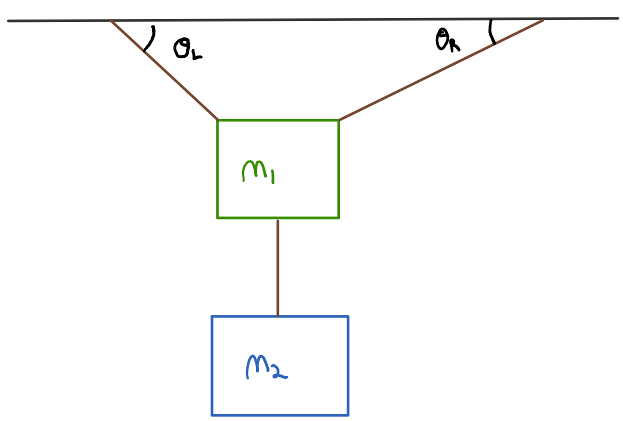
$|\vec F^N_{A1}| - m_1 g - |\vec F^P| \sin(\theta) = 0$

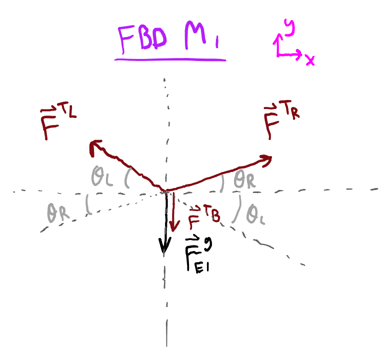
$\Sigma F_x = m_1 a_x$

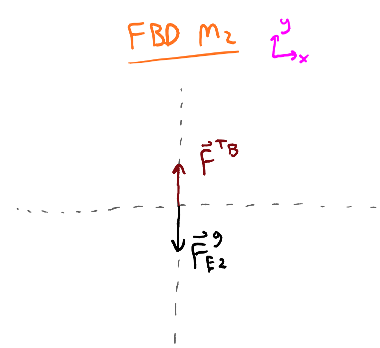
$F^P_x = m_1 a_x$

$|\vec F^P| \cos(\theta) = m_1 a_x$

PRACTICE: Two boxes are hanging from a ceiling as shown in the figure below. Draw a FBD for each mass. Attempt to scale each force relative to the others. Then translate the FBD into equations using Newton's 2nd law. *can you think of another FBD you can construct for this scenario?*







$\Sigma F_x = m_1 \cancel{a_{1x}}^{0}$

$|\vec F^{T_R}| \cos(\theta_R) - |\vec F^{T_L}| \cos(\theta_L) = 0$

$\Sigma F_y = m_1 \cancel{a_{1y}}^{0}$

$|\vec F^{T_L}| \sin(\theta_L) + |\vec F^{T_R}| \sin(\theta_R) - |\vec F^{T_B}| - |\vec F^g_{E1}| = 0$

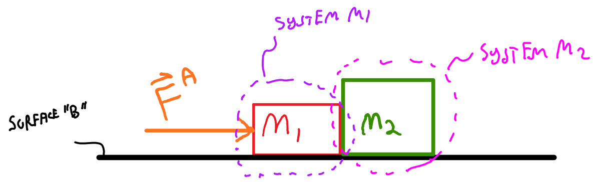
$|\vec F^{T_L}| \sin(\theta_L) + |\vec F^{T_R}| \sin(\theta_R) - |\vec F^{T_B}| - m_1 g = 0$

$\Sigma F_y = m_2 \cancel{a_{2y}}^{0}$

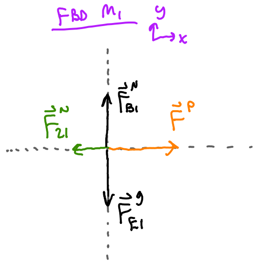
$|\vec F^{T_B}| - |\vec F^g_{E2}| = 0$

$|\vec F^{T_B}| - m_2 g = 0$

PRACTICE: Two boxes are on a frictionless table being pushed by some applied force as shown in the figure below. Draw a FBD for each mass below. Attempt to scale each force relative to the others. Then translate the FBD into equations using Newton's 2nd law



FBD m_1

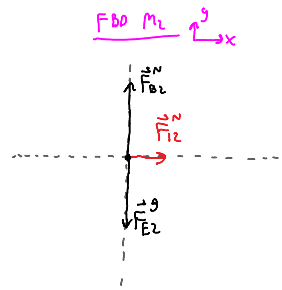


CONNECTIONS

$a_{1x} = a_{2x} \equiv a_x$

$|\vec{F}^N_{21}| = |\vec{F}^N_{12}|$

FBD m_2



$\Sigma F_x = m_1 a_{1x}$

$|\vec{F}^P| - |\vec{F}^N_{21}| = m_1 a_x$

$\Sigma F_y = m_1 \cancel{a_{1y}}^{0}$

$|\vec{F}^N_{B1}| - |\vec{F}^g_{E1}| = 0$

$|\vec{F}^N_{B1}| - m_1 g = 0$

$\Sigma F_x = m_2 a_{2x}$

$|\vec{F}^N_{12}| = m_2 a_x$

$\Sigma F_y = m_2 \cancel{a_{2y}}^{0}$

$|\vec{F}^N_{B2}| - |\vec{F}^g_{E2}| = 0$

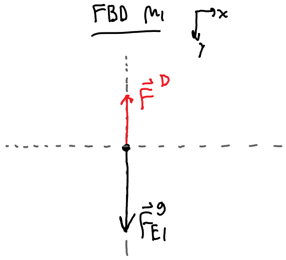
$|\vec{F}^N_{B2}| - m_2 g = 0$

PRACTICE: Draw a FBD for the situations below. Attempt to scale each force relative to each other. Then translate the FBD into equations using Newton's 2nd law.

1. A ball is falling vertically downwards towards earth and is speeding up. Assume that there is an air resistance force that is proportional to the speed of the ball and points in the opposite direction of motion.

$|\vec{F}^D| \propto |\vec{v}|$

FBD m_1



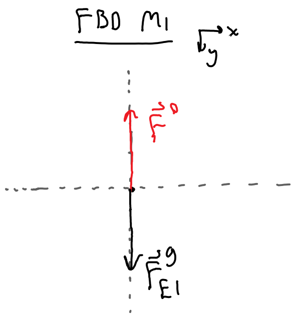
$\Sigma F_y = m_1 a_{1y}$

$-|\vec{F}^D| + |\vec{F}^g_{E1}| = m_1 a_{1y}$

$-k|\vec{v}| + m_1 g = m_1 a_{1y}$

a_{1y} DEPENDS ON SPEED .... i.e. THE ACCELERATION IS NOT CONSTANT ... NEED CALCULUS TO FIND OUR TYPICAL KINEMATIC VARIABLES

2. A ball is falling vertically downwards toward Earth and is not speeding up or slowing down, (it has reached terminal velocity). Assume that there is an air resistance force that is proportional to the speed of the ball squared and points in the opposite direction of motion.



$$\Sigma F_y = m_1 \cancel{a_{1y}}^{\,0}$$

$$-|\vec{F}^D| + |\vec{F}^g_{E1}| = 0$$

$$-D|\vec{v}|^2 + m_1 g = 0$$

$$|\vec{v}| = \sqrt{\frac{m_1 g}{D}}$$ ~ TERMINAL "VELOCITY"

Conceptual questions for discussion

1) When discussing Newton's second law with your friend, your friend says, "Force is equal to mass times acceleration, so each force acting on an object causes an acceleration and the net acceleration of the object is found by vector addition". Do you agree with this statement? Do you not? What would you tell your friend and why?
2) When summing the forces on your FBD, the direction of the coordinate system is not important. Discuss this statement.