LECTURE 17: Mechanical advantage

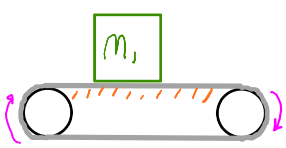
Select LEARNING OBJECTIVES:

- Introduce the concept of mechanical advantage, and hint at energy conservation via forces over distances.
- Be able to identify a system which provides mechanical advantage.
- Introduce the massless frictionless pulley, specifically how these pulleys can be configured to increase or decrease mechanical advantage.
- Be able to design a system that provides mechanical advantage.
- Be able to calculate mechanical advantage based on a FBD.
- Know how to calculate the total mechanical advantage of coupled systems, each contributing mechanical advantage to the total.
- Be able to identify when two objects are coupled or uncoupled based off of the relative motion between the two objects.
- Be able to identify the constraints that arise from coupled systems. For example, is $a_{1x}=a_{2x}$ or $a_{1x}=2a_{2x}$.

TEXTBOOK CHAPTERS:

- Giancoli (Physics Principles with Applications 7th) :: 4-7
- Knight (College Physics : A strategic approach 3rd) :: N/A
- BoxSand :: Forces (Mechanical Advantage)

WARM UP: Your luggage rides along top of a conveyor belt that is speeding up as seen in the figure below. If the luggage is not sliding across the surface of the conveyor belt, what type of friction is present and in which direction does it point?



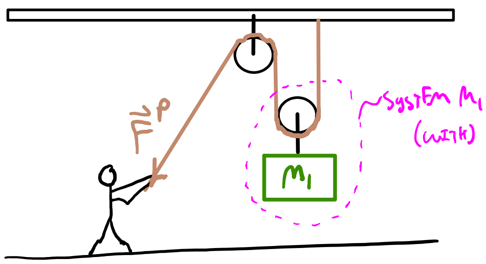
We have already seen that when we couple objects together in certain ways, we can scale the kinematic variables of each object relative to the other. That is to say, we can create systems where for every 1 meter object A moves, object B will move 2 meters. This type of coupling is of interest to many people. In the above example, if a person needed to move object B by 2 meters, the person could just move object A by 1 meter and ta da, object B magically moved the desired 2 meters! To easily quantify systems that provide such a coupling, we use what is known as mechanical advantage. Mechanical advantage is defined as the ratio of the amount of force required to do a task without a device to the amount of force required to do the task with a device. It is basically a measure of how much "easier" it is to complete a task with a device than without a device. A device can be anything, ranging from pulleys, to inclines, levers, etc… However, in this lecture, we will focus only on pulleys and introduce the other devices later on. The definition of mechanical advantage (MA) in terms of magnitude of force as shown below.

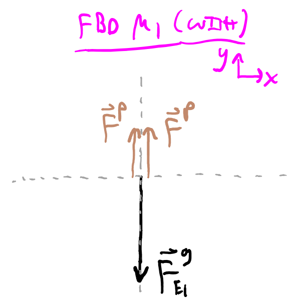
$MA = |\vec{F}|_{(\text{WITHOUT})}$

$$MA = \frac{|\vec{F}|_{(\text{WITHOUT})}}{|\vec{F}|_{(\text{WITH})}}$$

The best way to determine MA is to draw and analyze free body diagrams for the system with the device, and again without the device.

EXAMPLE: Determine the mechanical advantage of the pulley system below.

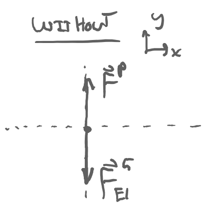




$$\Sigma F_y = M_1 a_{1y}^{\,0}$$

$$2|\vec{F}^P| - |\vec{F}^g_{E1}| = 0$$

$$|\vec{F}^P| = \tfrac{1}{2} M_1 g$$



$$\Sigma F_y = M_1 a_{1y}^{\,0}$$

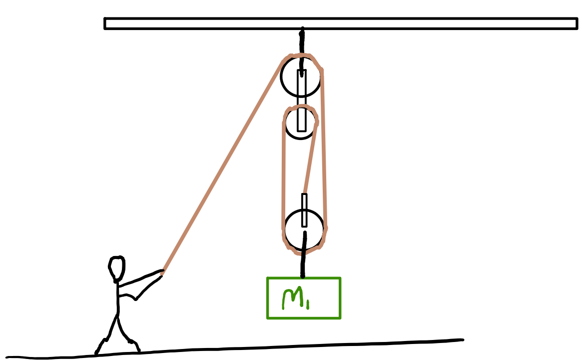
$$|\vec{F}^P| - |\vec{F}^g_{E1}| = 0$$

$$|\vec{F}^P| = M_1 g$$

$$MA = \frac{|\vec{F}^P|_{\text{WITHOUT}}}{|\vec{F}^P|_{\text{WITH}}}$$

$$MA = 2$$

PRACTICE: Determine the mechanical advantage of the pulley system below.



Questions for discussion

1) Is the mechanical advantage of a pulley system always equal to the number of pulleys in the system?
2) Connecting pulleys in such a way to create mechanical advantage seems impossible. Doesn't it feel like some sort of law of physics is being violated? After all, how can you only need a tiny bit of force to lift a massive object after connecting some pulleys? It turns out no laws are broken and we will address this unsettling feeling later.
    a. For now, examine the example problem in this lecture. If the person wanted to lift the box 1 meter upwards, how far must they pull the rope?
    b. Now look at the box, as the person pulls it up 1 meter, how much force was acting on this box during this 1 meter trip?
    c. Do you notice anything interesting about parts a and b? Think about products of forces and distances.