LECTURE 18: Fluid dynamics - Bernoulli's equation

Select LEARNING OBJECTIVES:

- Apply previous knowledge about conservation of energy to derive Bernoulli's equation
- Understand how increases or decreases in fluid speed affects pressure.
- Strengthen the ability to solve simultaneous equations.

TEXTBOOK CHAPTERS:

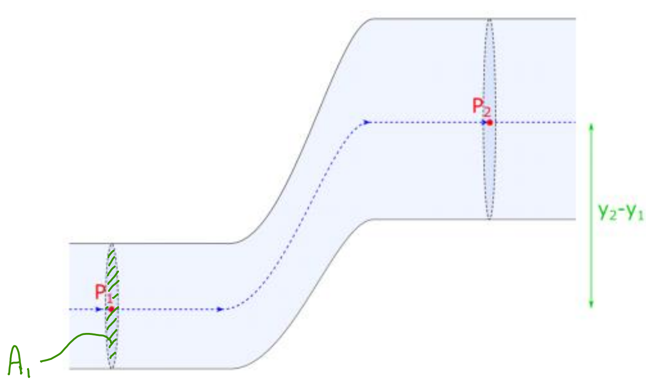
- Ginacoli ((Physics Principles with Applications 7th) :: 10-9, 10-10
- Knight (College Physics : A strategic approach 3rd) :: 13.6
- Boxsand :: Bernoulli's Principle

WARM UP: Is oil a fluid that we might consider studying with our current model of fluid dynamics?

This section will introduce perhaps the most fundamental principle in our studies of fluid mechanics, Bernoulli's principle. It is a direct application of conservation of mechanical energy and proves useful for both fluid dynamics and fluid statics. Bernoulli's equation and the continuity equation are usually the two most useful tools when approaching a fluid mechanics problem.

Bernoulli's equation relates the speed, relative height, and pressure at any location within a moving or stationary fluid. Bernoulli's equation and the continuity equation are usually the two most useful tools when approaching a fluid mechanics problem. Before defining Bernoulli's equation, a simple derivation will help demystify its appearance.

Consider the figure below which shows a moving fluid inside a tube that changes height and radius. We will consider an ideal fluid, (i.e. laminar flow, no viscous forces, and incompressible). The pressure is measured at the center of each cross sectional area as indicated by the red P_1 and P_2. The dashed blue line indicates the stream line that passes through the two locations we will consider.



Let's apply conservation of mechanical energy to this system (fluid + earth).

$$KE_i + U_i^g + \cancelto{0}{U_i^s} + \cancelto{0}{W_{INT}^{NC}} + W_{EXT} = KE_f + U_f^g + \cancelto{0}{U_f^s}$$

NO SPRINGS IN SYSTEM

NO INTERNAL FRICTION "IDEAL FLUID"

$$KE_i + U^g_i + W_{Ext} = KE_f + U^g_f$$

$$\tfrac{1}{2} m_f v_1^2 + m_f g y_1 + |\Sigma \vec{F}| \Delta x \cancel{\cos\theta}^{\,1} = \tfrac{1}{2} m_f v_2^2 + m_f g y_2$$

FORCE IS DUE TO PRESSURE DIFFERENCE BETWEEN THE TWO LOCATIONS

$$\tfrac{1}{2} m_f v_1^2 + m_f g y_1 + \Delta P_{12} A \Delta x_{12} = \tfrac{1}{2} m_f v_2^2 + m_f g y_2$$

CROSS SECTIONAL AREA TIME Δx_{12} IS THE VOLUME OF THE FLUID $\equiv V_f$

$$\tfrac{1}{2} \rho_f \cancel{V_f} v_1^2 + \rho_f \cancel{V_f} g y_1 + \Delta P_{12} \cancel{V_f} = \tfrac{1}{2} \rho_f \cancel{V_f} v_2^2 + \rho_f \cancel{V_f} g y_2$$

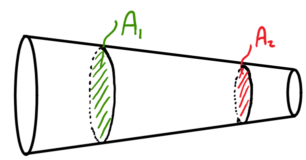
$$\tfrac{1}{2} \rho_f v_1^2 + \rho_f g y_1 + (P_1 - P_2) = \tfrac{1}{2} \rho_f v_2^2 + \rho_f g y_2$$

$$\boxed{P_1 + \tfrac{1}{2} \rho_f v_1^2 + \rho_f g y_1 = P_2 + \tfrac{1}{2} \rho_f v_2^2 + \rho_f g y_2}$$

* BERNOULLI'S EQUATION

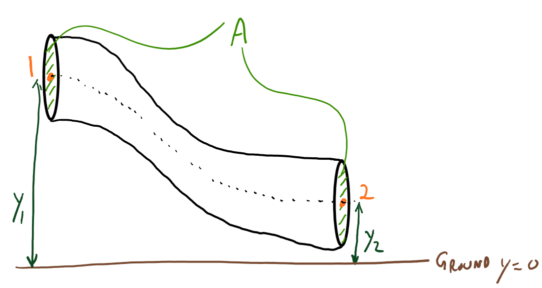
Note that each term has units of energy per volume, thus this is a now a statement of conservation of mechanical energy density. Also note that the two locations we picked were arbitrary, thus what Bernoulli's equation is telling us is that some quantity (units of energy density) is a constant at all locations within our ideal fluid.

PRACTICE: Water flows through a horizontal pipe as shown in the figure below. What can be said about the pressure at points 1 and 2?



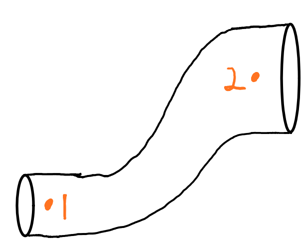
(a) $P_1 > P_2$
(b) $P_1 < P_2$
(c) $P_1 = P_2$
(d) None of the above.

PRACTICE: Water flows through a constant diameter pipe as shown in the figure below. What can be said about the pressure at points 1 and 2?



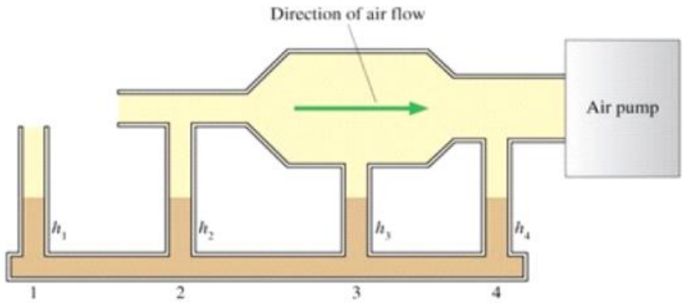
(a) $P_1 > P_2$
(b) $P_1 < P_2$
(c) $P_1 = P_2$
(d) P_1 could be less than or greater than P_2 depending on y.

PRACTICE: Water flows from 1 to 2 in a pipe system like in the figure below. The cross-sectional area at point 1 is less than at point 2 and point 2 is higher in elevation than point 1. Which of the following statements are plausible. (drawing not to scale)

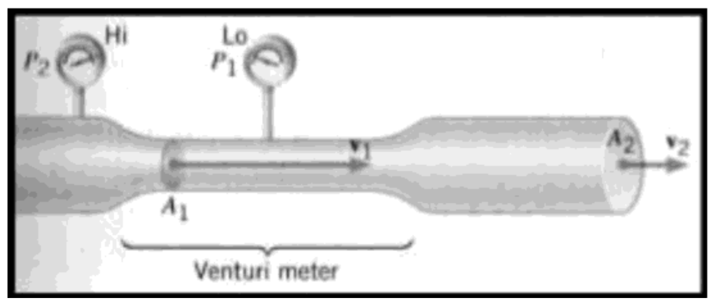


(a) The speed of the water at point 1 is less than at point 2.
(b) The speed of the water at point 1 is equal than at point 2.
(c) The speed of the water at point 1 is greater than at point 2.
(d) The pressure at point 1 is greater than at point 1.
(e) The pressure at point 1 is equal to that at point 2.
(f) The pressure at point 2 is less than at point 2.

PRACTICE: Rank in order, from highest to lowest, the liquid heights h_1 to h_4 in tubes 1 to 4. The air flow is from left to right.



PRACTICE: A Venturi meter is a device for measuring the speed of fluid within a pipe. The drawing shows a gas flowing at a speed v_2 through a horizontal section of pipe whose cross-sectional area is $A_2 = 0.0700\ m^2$. The gas has a density of $1.30\ kg/m^3$. The Venturi meter has a cross-sectional areas of $A_1 = 0.050\ m^2$ and has been substituted for a section of the larger pipe. The pressure difference between the two sections is $P_2 - P_1 = 120$ Pa. Find (a) the speed v_2 of the gas in the larger original pipe and (b) the volume flow rate Q of the gas.



QUESTIONS FOR DISCUSSION:

(1) When a baseball is thrown in such a way that the ball curves to the right, air is flowing
   a. faster over the left side than over the right side.
   b. faster over the right side than over the left side.
   c. faster over the top than underneath.
   d. at the same speed all around the ball, but the ball curves as a result of the way the wind is blowing over the field.

(2) A car with a fabric convertible top is driving down the highway with the windows closed and the top up. Why does the fabric top bulge outwards when traveling at high speeds?

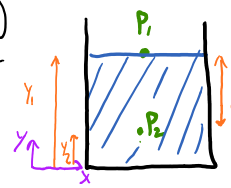
(3) Children are often told to avoid standing too close to a rapidly moving train because they might get "sucked" under it. Is this possible? Explain.

Hydrostatics?

Look at Bernoulli's equation carefully. Notice the term with the speed squared? Any new model should also incorporate previous well established models that exist. For instance, we previously derived an expression for pressure at a depth of static (non-moving) fluids. Thus as the speed approaches zero we should expect Bernoulli's equation to not violate any hydrostatics relationships we have discovered before. Basically, if the speed of the fluid is very slow we should expect the same results as pressure at a depth. We can check to see if this is the case by setting the speed equal to zero in Bernoulli's equation. And indeed, we get our familiar pressure at a depth expression. Now you know that the only two equations you will need for fluid mechanics (hydrostatics and dynamics) are the continuity equation and Bernoulli's equation, both of which resulted from conservation laws.

PRESSURE AT A DEPTH (HYDRO STATICS)

$$P_2 = P_1 + \rho_F g d$$



BERNOULLI'S EQUATION w/ $v \rightarrow 0$

$$P_1 + \tfrac{1}{2}\rho_F \cancelto{0}{v_1^2} + \rho_F g y_1 = P_2 + \tfrac{1}{2}\rho_F \cancelto{0}{v_2^2} + \rho_F g y_2$$

$$P_1 + \rho_F g y_1 = P_2 + \rho_F g y_2$$

$$P_2 = P_1 + \rho_F g y_1 - \rho_F g y_2$$

$$P_2 = P_1 + \rho_F g \underbrace{(y_1 - y_2)}_{d}$$

$$P_2 = P_1 + \rho_F g d \checkmark$$