LECTURE 22: Statics

Select LEARNING OBJECTIVES:

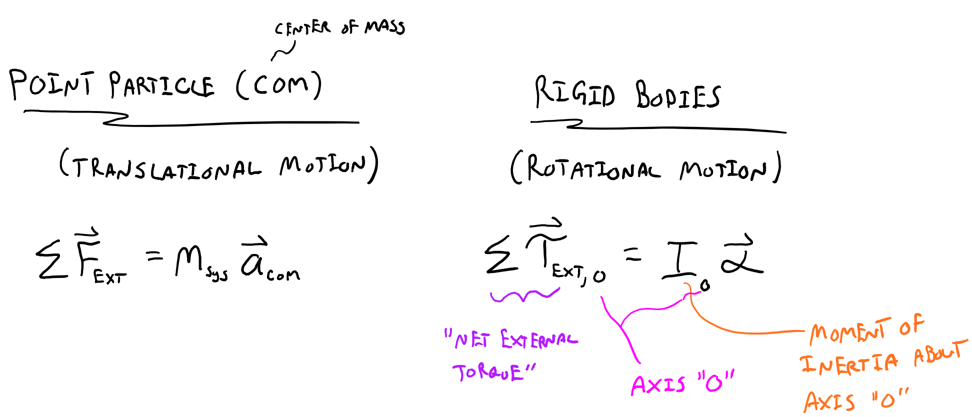
i. Define cross product and be able to demonstrate an understanding of its application through the use of the mathematical representation and physical representation.
ii. Define torque, and demonstrate an understanding of its functional dependence on the magnitude of a force, the moment arm, and the angle between the two.
iii. Understand that torque is a vector even though we only use a sign convention to describe its direction.
iv. Demonstrate the ability to determine the net torque on a rigid body through the use of a physical representation coupled with a mathematical representation.

TEXTBOOK CHAPTERS:

- Giancoli (Physics Principles with Applications 7th) :: 9-1, 9-2, 9-3, 9-4
- Knight (College Physics : A strategic approach 3rd) :: 7.4, 8.1, 8.2
- BoxSand :: Rotational Mechanics (Statics & Dynamics)

WARM UP: Does the reference axis "o" have to be the actual pivot point that an object rotates about?

Now that we have defined a way to mathematically work with torque, let's revisit Newton's laws of motion as applied to the point particle model (linear motion) and also introduce its form in the rigid body model (rotational motion).



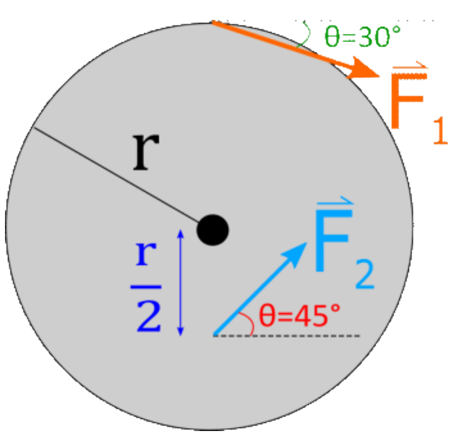
Statics and dynamics are terms used to describe the different scenarios that fall out of the two versions of Newton's 2nd law above. The scenarios can be summarized by the table below. In this section we will focus mostly on the rotational portion since this is the newer topic being introduced.

	Translational	Rotational
Static Equilibrium	$\vec{v}_{com}=\vec{0}$	$\omega=0$
Dynamic Equilibrium	$\vec{v}_{com}=\text{constant}\neq\vec{0}$ $\vec{a}_{com}=\vec{0}$ $\Sigma\vec{F}=\vec{0}$	$\omega=\text{constant}$ $\alpha=0$ $\Sigma\tau_o=0$
Dynamics	$\Sigma\vec{F}=m\vec{a}_{com}$	$\Sigma\tau_o=I_o\alpha$

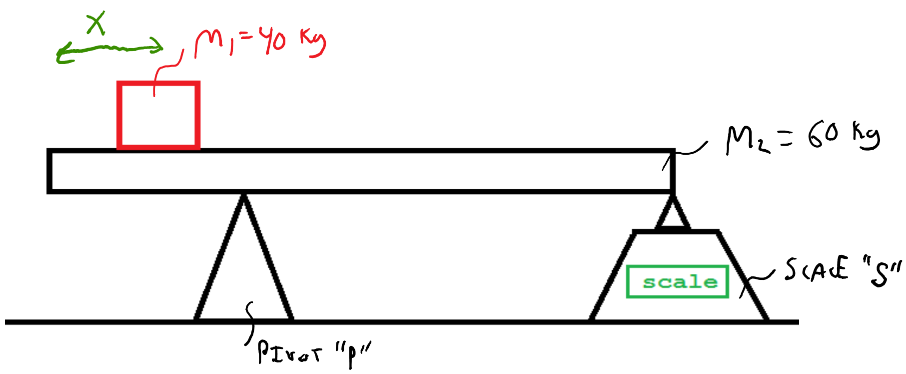
A new quantity shows up when dealing with rotational dynamics, moment of inertia I_o, where the subscribe "o" refers you to the reference axis of rotation. The moment of inertia is analogous to mass in the point particle model. The mass in the point particle model plays the role of inertia, which we can view in the following way: the larger the mass (i.e. the larger the inertia) the harder it is to increase the translation motion of an object. Think about a massive object on a frictionless surface, if you stand on a surface with friction and push the massive object, it is really hard to start to get moving at a high speed. If the object were much less massive, you would have no problem getting the object up to a fast speed. Now we can understand what role the moment of inertia plays in rotational motion. Just like mass, the larger the moment of inertia, the harder it is to increase the rotational motion of an object about an axis. Again, a simplified thought experiment goes something like this: if object that is not rotating has a large moment of inertia, it would require a lot of effort on your part to get it to start rotating at a fast angular speed. If the object has a much smaller moment of inertia, it would be much easier to get it rotating at a fast speed.

The moment of inertia depends on the reference axis of rotation and the way that the mass is distributed around this reference axis of rotation. For example, if you hold weights in our hands and stretch your arms out you would have a larger moment of inertia about the axis that runs from your feet to your head than you would if you held the weights tight to your body.

PRACTICE: The figure below shows two forces acting on a uniformly distributed 7 kg solid disk that is free to rotate about its center and has a diameter of d=3.0 cm. What must the magnitude of force 2 be such that the disk is in rotational static equilibrium? The moment of inertia for a disk about its center, perpendicular to the face of the disk, is $1/2\ m\ r^2$, where r is the radius. Let the magnitude of force 1 be equal to 10 N.



PRACTICE: A table top of mass 60 kg which is uniformly distributed is 2.4 m long and is supported by a pivot 0.8m from the left end, and by a scale at the right end.



1. How far from the left end should a 40 kg box be placed on the table top if the scale is to read 100 N?

1. With the box at this location, what is the normal force provided by the pivot?

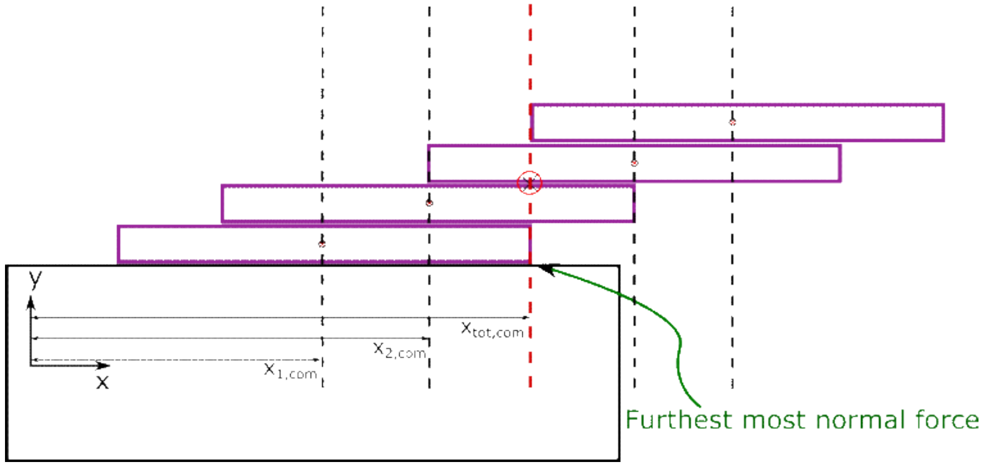
Stability

Often times summing the torque of a system of objects around an axis to determine if the system is in equilibrium or not is tedious. It turns out we also have another method to determine the stability of a system by looking at the total center of mass of the system and its location to the furthest most normal force.

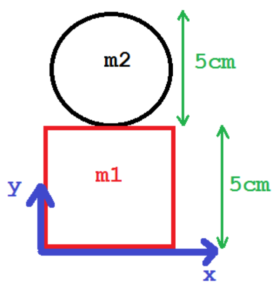
Mass is distributed within a rigid body, however, we can define a "center of mass" (com) location for an object or system of objects. This is useful because if our object is in a uniform gravitational field (much like the approximate uniform gravitation field near the surface of the Earth), then the force of gravity that acts over the entire body of the object can be simplified and treated as if it acts through only the center of mass of the object. The center of mass will have a x, y, and z-component, but often times we can just consider the x and y-component because the z-component is of no importance to our problem. Mathematically we find the center of mass of a system of objects as follows

$$X_{com}^{TOT} = \frac{X_{com}^{1} M_1 + X_{com}^{2} M_2 + X_{com}^{3} M_3 + \ldots}{M_1 + M_2 + M_3 + \ldots} = \frac{\sum X_{com}^{i} M_i}{\sum M_i}$$

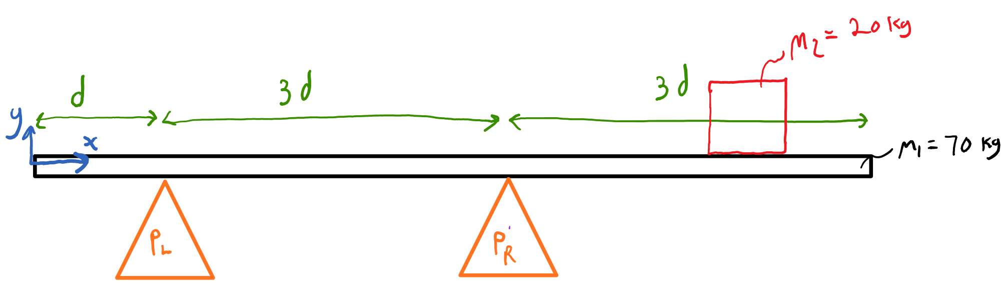
$$Y_{com}^{TOT} = \frac{y_{com}^{1} M_1 + y_{com}^{2} M_2 + y_{com}^{3} M_3 + \ldots}{M_1 + M_2 + M_3 + \ldots} = \frac{\sum y_{com}^{i} M_i}{\sum M_i}$$



PRACTICE: A solid cylinder sits atop a solid cube as shown below. Both objects masses are uniformly distributed. What is the center of mass of the combined system? Let m1 = 0.8 kg and m2 = 0.4 kg.



PRACTICE: A 70 kg plank lies on top of two triangular supports. What is the furthest distance, measured from the left side of the plank, that a 20 kg mass could lie without tipping the plank over?



Questions for discussion:

(1) Discuss the validity of the statement: "torque keeps objects rotating".
(2) Discuss the validity of the statement: "if something is not rotating, it has a net torque of zero".
(3) Can a ruler be in equilibrium at an angle relative to the horizontal?