LECTURE 24: Conservation of momentum
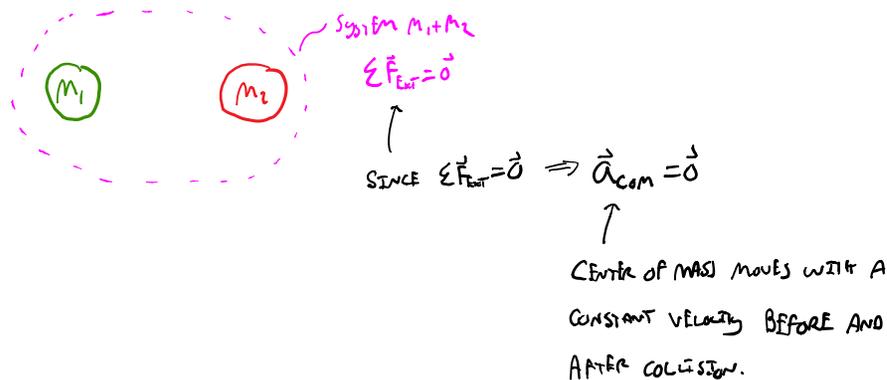
Select LEARNING OBJECTIVES:

    i.   Introduce the definition of momentum conservation.
    ii.   Understand the conditions for which momentum is conserved, (i.e. no net external forces).
    iii. Introduce center of mass.
    iv. Be able to calculate the center of mass of different geometrical objects and system of objects.
    v.   Demonstrate the ability to properly define a system and apply momentum conservation.
    vi. Demonstrate the ability to read a problem, and then reflect upon what information is given to be able to determine what type of physics is relevant to the problem.
    vii. Strengthen the ability to perform vector addition and subtraction.

TEXTBOOK CHAPTERS:

- Giancoli (Physics Principles with Applications 7th) :: 7-2
- Knight (College Physics : A strategic approach 3rd) :: 9.4
- BoxSand :: Momentum (Conservation of Momentum)

WARM UP: Use this (https://phet.colorado.edu/sims/collision-lab/collision-lab_en.html) simulation to study the center of mass carefully before, during, and after the collision. Based off of your observations of the center of mass, what statements can you make?

System $M_1 + M_2$

$\Sigma \vec{F}_{Ext} = \vec{0}$

SINCE $\Sigma \vec{F}_{Ext} = \vec{0} \Rightarrow \vec{a}_{Com} = \vec{0}$

CENTER OF MASS MOVES WITH A CONSTANT VELOCITY BEFORE AND AFTER COLLISION.

Recall the impulse-momentum theorem…

$$\vec{J} \equiv \Delta \vec{p} = \Sigma \vec{F}_{Ext} \Delta t$$

Notice the right hand side, if the net external force acting on the system is very small, or if the time that the net

external force is acting on the system is very small, then the change in momentum is approximately zero as seen below.

IF $\sum \vec{F}_{ExT} \approx \vec{0}$ OR   IF $\Delta t \approx 0$ OR   IF $\Delta t \approx 0$ AND $\sum \vec{F}_{ExT} \approx \vec{0}$

THEN $\Delta \vec{p} = \sum \vec{F}_{ExT} \Delta t$

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$\Delta \vec{p} = \sum \vec{F}_{ExT} \Delta t$

$\Delta \vec{p} \approx \vec{0}$

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Is this a valid approximation? It turns out, if we are careful with defining our system, we can often make use of this approximation. For example, consider two billiard balls colliding into each other. The normal force from billiard ball 1 acts on billiard ball 2, this changes the momentum of billiard ball 2. Likewise, via Newton's 3rd law, the normal force from billiard ball 2 on billiard ball 1 changes the momentum of billiard ball 1. If we choose our system to include both the billiard balls, then these normal forces are internal forces as seen below.
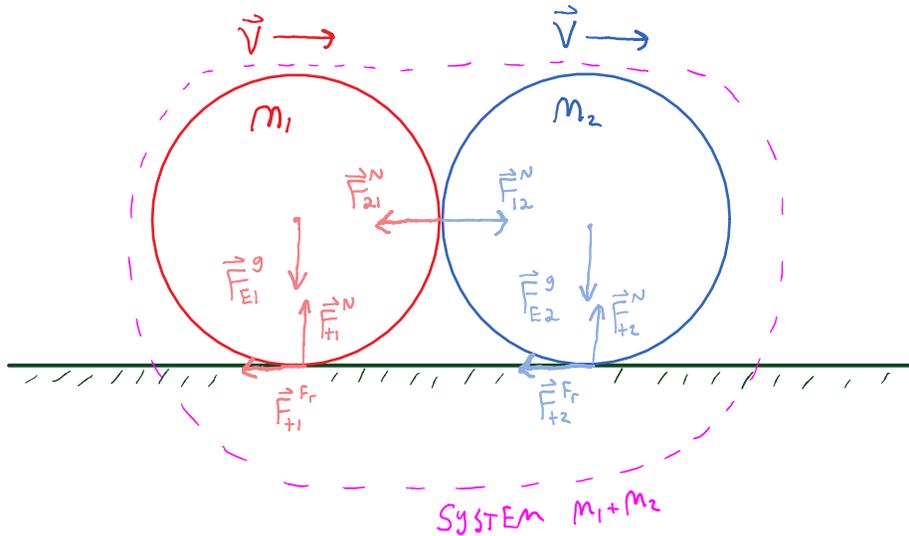
However there are other forces are acting on our system, the normal force from the table on each billiard ball, the force of gravity on each ball and friction between the table and each ball. The normal forces form the table and the force of gravity cancel each other out, so there is no longer a net force in the vertical direction. This just leaves friction left as an external force. However, think about billiard balls colliding, how much time are they in contact with one another? It is a very very short time. Not only is the time interval very small during collisions of billiard balls, the friction (which is due to rolling friction) is also very small between billiard balls and pool tables (if you don't believe me, roll a billiard ball at some speed and notice it takes a relatively long time for the rolling friction to slow it down, thus the rolling friction is small). Now the net external force for our billiard ball collision example is reduced to a small value for friction which acts during a small amount of time when the collision occurs, thus we can make use of the approximation that the change in momentum is zero. This is why physicists love using billiard balls as objects when talking about collisions.

Continuing on with our billiard ball example, we now have the expression…

$$\Delta \vec{p}_{Sys} = \sum \underset{on\ sys}{\vec{F}_{ExT}} \Delta t$$

$\approx \vec{0} \qquad \approx 0$

$$\Delta \vec{P}_{sys} = \Sigma \vec{F}_{\substack{Ext \\ on \, sys}} \Delta t$$

$$\Delta \vec{P}_{sys} = \vec{0}$$

This change in momentum of our system can be rearranged...

$$\vec{P}_{f_{sys}} - \vec{P}_{i_{sys}} = \vec{0}$$

$$\vec{P}_{i_{sys}} = \vec{P}_{f_{sys}}$$

$$\Sigma \vec{P}_i = \Sigma \vec{P}_f$$

...where we notice that our system is comprised of multiple objects (in the billiard ball example, ball 1 and ball 2). Thus the momentum of our system is just the sum of the momentum for each object included in our system. Take a momentum to appreciate what just happened. We started from the impulse-momentum theorem, defined a system for which the normal forces between objects colliding are internal forces, then because our net external force and collision time interval were very small, the initial momentum of our system is equal to our final momentum of our system. This is known as conservation of momentum, and is a general statement for which our approximations are valid, or if the net external force is zero. When the net force on a system is zero, it is often called an isolated system.

CONSERVATION OF MOMENTUM

$$\boxed{\Sigma \vec{P}_i = \Sigma \vec{P}_f}$$

During a collision, we could measure all the net forces (and approximate any time dependent forces with a constant average force), draw a free body diagram, use Newton's laws of motion to find the acceleration of each object, then use kinematics with the measured time interval of the collision to find final velocities if the initial velocities were known. This is quite the complex problem, but it is doable. To avoid this mess, the first thing you want to do is identify a system where you can use conservation of momentum, then you can tackle these tricky problems with relative ease! The next lecture will introduce how we can use conservation of momentum for various types of scenarios.

Problems for discussion

1) Can you think of a scenario where the net external force on a system during a collision would be exactly zero?
2) When to cars collide you decide to define your system to include both cars. Is your system isolated?