LECTURE 26: Work- Kinetic Energy

Select LEARNING OBJECTIVES:

i. Introduce and define linear kinetic energy.
ii. Strengthen the ability to perform proportional reasoning through the use of kinetic energy related questions.
iii. Introduce the relationship between kinetic energy and momentum.
iv. Introduce the concept of work through the work-energy theorem.
v. Demonstrate the ability to mathematically and graphically calculate the dot product between two vectors.
vi. Be able to determine the sign of work if given a physical representation, graphical representation, or written description.
vii. Be able to analyze a force vs displacement graph, relating area to work.
viii. Be able to differentiate between when to use momentum and when to use energy.
ix. Be able to apply the work-energy theorem to analyze problems involving forces acting on objects over a distance.

TEXTBOOK CHAPTERS:

- Giancoli (Physics Principles with Applications 7th) :: 6-1, 6-2, 6-3
- Knight (College Physics : A strategic approach 3rd) :: 10.2, 10.3
- BoxSand :: Energy (Work & Kinetic Energy)

WARM UP: No warm up this week. Read Feynman’s introduction to energy found in the “other” section of the calendar for this week.

Energy is...um, well...energy can be described as...we can think of energy as... Ok so, it is no consequence that I cannot write a short sentence here explaining what energy is. I can tell you that it comes in many different forms. For example, let's consider motion of an object in 1D. One of the kinematic equations that describes 1-D motion by relating different quantities in a mathematical form is...

$$v_f^2 = v_i^2 + 2a_x \Delta x$$

Also recall Newton’s 2nd law of motion...

$$F_{ix} = m a_x$$

A simple substitution of Newton’s 2nd law into the kinematic equation above, and some simple algebraic rearrangements yields the following...

$$\frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2 = \int F_{ix} \Delta x$$
From the right hand side of the above equation, it appears that forces acting over a distance results in some sort of change in velocity as described on the left hand side of the equation. And as it turns out, this manipulation of equations will be of great benefit when trying to analyze the motion of objects. In fact, this equation has new expressions that will be so important to solving problems, that we will define some new names for these quantities...

Using our newly defined terms, we can now write what we call, the "work-kinetic energy theorem", which takes the mathematical form below...

$$\sum W_{\text{ext}} = \Delta KE$$

This expression above outlines the basics of what the "work-kinetic energy theorem" describes, namely that a system's change in kinetic energy is due to the net external work on a system by its environment. This helps clarify what work is; work is a mechanism for which interacting objects exchange energy. Nothing possess work, thus we would not say "a box has 10 J of work", we could say something like "the box lost 5 J of energy to its environment via work done on the box by the environment". This form is often rearranged in the following way...

$$KE_f - KE_i = \sum W_{\text{ext}}$$

This interpretation of the work-energy theorem states that "take the initial kinetic energy of the system and add it to the net external work being done on the system, this addition is equal to the final kinetic energy of the system".

It is very important to note, that energy and work are both scalars. Energy and work are scalars. The above "derivation" of the work-kinetic energy theorem was done in one dimension, but even in two or three dimensions, energy and work are both scalars.

*Defining a system will still be important when using an energy analysis method. However, there are some confusing subtleties that we didn't need to deal with when looking at systems for force and momentum analysis methods. For this lecture, we will only consider a system consisting of a single object. In the following lectures we will expand this simplification to include systems that contain multiple objects.
Recall kinetic energy is a scalar, thus work must also be a scalar. To calculate work we use what is known as the "scalar product" or dot product.

\[ W = \vec{F} \cdot \Delta \vec{r} \]

This is only the definition of work for a constant force.

\[ W = |\vec{F}| |\Delta \vec{r}| \cos(\theta) \]

Where \( \vec{F} = |\vec{F}| \cos(\theta) \) and \( \Delta \vec{r} = |\Delta \vec{r}| \cos(\theta) \).

Additionally, if we consider a 1-D scenario where a force acts either parallel or anti parallel to the direction of displacement, and we let the direction of motion be along the x-axis, then work is just the magnitude of the force (for 1-D this is just the x-component of force) times the magnitude of displacement (just \( \Delta x \) for 1-D) with either a plus or minus one depending on their relative directions.

\[ W = F_x \Delta x \]

With a definition like the one above, we should be comfortable with the idea that if we plot \( F_x \) vs \( x \) we can graphically calculate work by finding the area under the curve as shown below.
EXAMPLE: A box is pushed across a horizontal frictionless table as shown in the figure below. What is the sign of work from all the forces acting on the box?

\[
\text{ANSWER UNDER } F_x \text{ VS } \text{ X-CUBE}
\]

\[2 \ J + \frac{1}{2}(\text{mass})(2 \ \text{m}) = \text{KE_f}
\]

\[6 \ J = \text{KE_f}
\]

UNITS... \( J = N \cdot m = \frac{\text{Kg} \cdot \text{m}^2}{\text{s}^2} \)

EXAMPLE: An object moving along the x-axis experiences a force shown in the graph below. If the object has 2.0 J of kinetic energy as it passes \( x = 0 \) m, what is its kinetic energy when it reaches \( x = 4 \) m?

PRACTICE: Which force does the most work? The displacement for each case is the same.
PRACTICE: A crane raises a steel girder into place at a construction site. The girder moves at a constant speed. For this problem use a system that includes just the girder. Consider the work done by the force of gravity ($W_g$) and the work done by the tension in the cable ($W_T$). Which of the following is correct?

1. $W_g$ and $W_T$ are both zero.
2. $W_g$ is negative and $W_T$ is negative.
3. $W_g$ is negative and $W_T$ is positive.
4. $W_g$ is positive and $W_T$ is positive.
5. $W_g$ is positive and $W_T$ is negative.

PRACTICE: A crane rises a steel girder into place at a construction site. The girder is moving downward and slowing down. For this problem use a system that includes just the girder. Which of the following statements about the girder are true?

(1) The kinetic energy is increasing.
(2) The kinetic energy is decreasing.
(3) The work from the cable is positive.
(4) The work from the cable is negative.
(5) The work from gravity is positive.
(6) The work from gravity is negative.
(7) The magnitude of the work from the cable is greater than that from gravity.
(8) The magnitude of work from the cable is less than that from gravity.
(9) The magnitude of the work from the cable is equal to that from gravity.

PRACTICE: A 2 kg block, initially at rest, is being pulled horizontally by a 5 N tension force. The coefficient of static and kinetic friction between the block and the table is 0.5 and 0.2, respectively. What are the sign of the following quantities?

(1) Work from gravity.
(2) Work from normal force.
(3) Work from tension.
(4) Work from friction.

Now the block is sliding in the direction of tension. What are the signs of the following quantities?

(1) Work from gravity.
(2) Work from normal force.
(3) Work from tension.
(4) Work from friction.
A cart on an air track is moving at 0.5 m/s when the air is suddenly turned off. The cart comes to rest after traveling 1 m. What is the coefficient of kinetic friction between the cart and the track?

Two people are on low friction carts, throwing a medicine ball back and forth, speeding up the rate at which they move away from each other. What is the sign of the work of the ball on person A while they catch the ball?

(1) Positive  
(2) Negative  
(3) Zero

What is the sign of the work of the ball on person A while they throw the ball?

(1) Positive  
(2) Negative  
(3) Zero
PRACTICE: The diagram depicts two pucks on a frictionless table. Puck II is four times as massive as puck I. Starting from rest, the pucks are pushed across the table by two equal forces. The forces are active all the way to the finish line.

Which puck will have a greater kinetic energy upon reaching the finish line?

PRACTICE: The diagram depicts two pucks on a frictionless table. Puck II is four times as massive as puck I. Starting from rest, the pucks are pushed across the table by two equal forces. The forces act on both of them for 6.0 s.

Rank the final kinetic energies of the two pucks.
PRACTICE: A pendulum bob swings in a complete vertical circle. At what point does the tension in the string do the most work?

(1) Bottom of the loop.
(2) Top of the loop.
(3) Tension does no work.

PRACTICE: A stuntman is sliding toward a brick wall with wind blowing down and against their motion. Strapped to their back is a rocket providing thrust.

What is the sign of the work for each force acting on the stuntman?

- Gravity
- Normal
- Friction
- Wind
- Thrust

PRACTICE: A gondola with a box inside is accelerating up and at an angle from the horizontal.

What is the sign of the following work?

1. Gravity on box
2. Gravity on gondola
3. Normal force on box
4. Normal force on gondola
5. Friction on box
6. Friction on gondola
7. Tension from cable on box
Questions for discussion:

(1) Can the work from kinetic friction on an object ever be positive? If so can you provide an example? If not, provide a few examples which support your claim.

(2) Can static friction ever do work (positive or negative) on an object? If so can you provide an example? If not, provide a few examples which support your claim.