LECTURE 36: Thin film interference

Select LEARNING OBJECTIVES:

- Be able to identify relative phase shifts and which conditional must be used.
- Be able to draw rays undergoing thin film interference.
- Be able to identify where the Path Length Difference occurs between the two interfering waves.
- Be able to find the series of wavelengths that will interference constructive/destructively.
- Be able to explain why we only consider the first two reflections.

TEXTBOOK CHAPTERS:

- Ginacoli ((Physics Principles with Applications 7th) :: 24-8
- Knight (College Physics : A strategic approach 3rd) :: 17.4
- Boxsand :: Thin film interference

WARM UP: Do we see a color change of light as it travels into a material with a greater index of refraction than air?

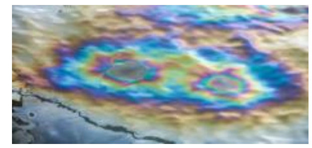
In this lecture we will conclude our exploration of interference effects from traveling waves. So far we have discussed quite a few interference situations. Before we dive into thin film interference, let's remind ourselves what we have discussed so far to help get a bearing of where we are. Below is a table with a brief review of some of the interference phenomena we have covered.

Identification	Overview	Type of interference	Constraints
Standing waves on a string	Create a wave traveling down a string which then reflects off of a boundary. As this reflected wave travels back to the source, send another wave down the string. The original reflected wave and the new traveling wave meet and interfere. Do this process continuously at the right frequencies and a standing waves form.	Spatial	• Continuous supply of waves traveling down the string. • Frequencies of standing waves could only take on discrete values based on the boundary conditions.
Standing sound waves in a pipe	Create a wave traveling sound wave traveling down a pipe which then reflects off of a boundary. As this reflected wave travels back to the source, send another sound wave down the pipe. The original reflected wave and the new traveling wave meet and interfere. Do this process continuously at the right frequencies and a standing waves form.	Spatial	• Continuous supply of waves traveling down the pipe. • Frequencies of standing waves could only take on discrete values based on the boundary conditions.
Beat frequencies	Two sources (e.g. two sound waves created with turning forks) with different frequencies produce traveling waves which interfere at a location in space. This interference at a fixed location in space fluctuates between constructive and destructive interference at a	Temporal	• Frequencies of two sources must be different.

	frequency referred to as the beat frequency.		
General two source interference	Two sources (e.g. two speakers) with the same frequency create traveling waves which produce interference patterns in space. At any one location in space there may be a completely constructive, completely destructive, or anywhere in-between interference condition.	Spatial	• Coherent sources. • Sources must have same frequency and wavelength.
Young's double slit apparatus	Use two slits to transmit a single source of light creating an interference pattern on a viewing screen far from the slits.	Spatial	• Coherent sources. • Sources have the same frequency and wavelength. • The distance (L) from the sources to the viewing screen must be much greater than the distance between the two sources (d). This is often referred to as the "far field approximation" or "Fraunhofer diffraction" . L >> d
Diffraction grating	Use many slits to transmit a single source of light creating an interference pattern on a viewing screen far from the slits.	Spatial	• Coherent sources. • Sources have the same frequency. • The distance (L) from the sources to the viewing screen must be much greater than the distance between the two sources (d). This is often referred to as the "far field approximation" or "Fraunhofer diffraction" . L >> d
Single slit	Use a single slit to transmit a single source of light creating an interference pattern on a viewing screen far from the slits.	Spatial	• Coherent source. • Source have the same frequency and wavelength. • The distance (L) from the sources to the viewing screen must be much greater than the distance between the two sources (d). This is often referred to as the "far field approximation" or "Fraunhofer diffraction" . L >> d
Thin film interference	Light wave reflects off two (or more) boundaries of a thin transparent medium. The reflected light can produce constructive and destructive interference patterns.	Spatial	• Coherent length of source must be on the order of the path length difference due to the thin film.

In general, the physics behind each interference phenomena we studied is governed by the same underlying physics; when waves meet they interfere mathematically by the superposition principle. Thus, you can really think of all of the interference phenomena as just different ways to create situations where waves can interfere. Thin film interference is no exception to this trend.

Thin film interference



When was the last time you created soap bubbles? Can you recall the wonderful colors that these bubbles produce. Or perhaps you noticed a similar assortment of colors on surfaces which have a thin layer of oil on them. These are two fantastic examples of thin film interference. When light shines on a very thin film of transparent media, interference can occur. We will construct a mathematical model for this thin film interference to help predict which wavelengths of light will constructively or destructively interference. Before we introduce the model we must first introduce some constrains and review how waves interact at boundaries.

Constraints

- Coherent length of source must be on the order of the path length difference due to the thin film.

  We were careful to use coherent sources for general two source interference and the multi-slit interference apparatuses. The reason is similar to beat frequencies; if the phase between the sources are constantly changing in time, then the interference pattern will also change in time washing out the spatial interference patterns we wished to observe. We can relax this constraint a little bit when working with thin films. The white light from the sun is not considered coherent, however the light from the sun does have some degree of coherence referred to as "coherence length". It is beyond the scope of this class, but important to point nonetheless. A good source for more information about coherence and coherence length can be found here.

- Simplification: We will only consider scenarios where light is normal to the thin film surface but we will draw diagrams at an angle for clarity. Do not get the diagram confused with what is physically happening. Below is a figure to help illustrate why we draw diagrams that do not match the physical phenomena.

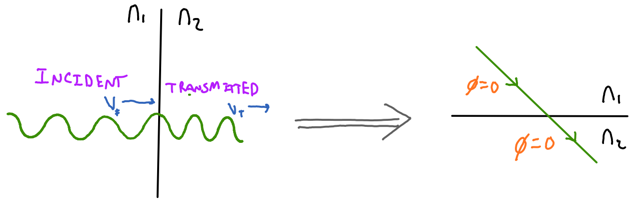
Thin film interference can also be modeled with the light is not normal to the thin film surface, the only added complexity is a sine or cosine term in the mathematical model which doesn't add any further insight into the physics behind the interference, thus we constrain ourselves to the simpler algebraic model when light is normal to the thin film surface.

Waves and boundaries review

Remember that any time a wave encounters a boundary some portion of the initial wave goes transmitted and some portion gets reflected. Since we are using the reflected waves off of a boundary to produce interference patterns, we must review and add to our discussion about waves reflecting from boundaries when we first introduced superposition in lecture 27. The most important piece of the puzzle when considering interference from reflected and possible transmitted light is the phase (ϕ) shifts a wave may possibly pick up when traversing boundaries between different media. Below are the three rules needed for determing how the phase (ϕ) shifts when a light wave encounters a different medium. Note that initial incident wave's phase is arbitrary, thus we will always set it to zero.

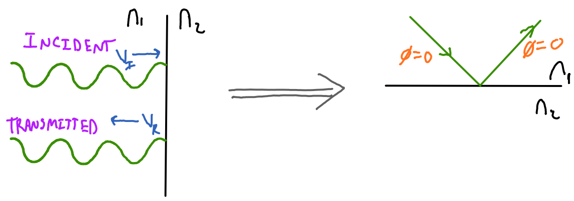
Transmitted light: ($n_1 > n_2$) OR ($n_1 < n_2$)

- No phase shift for transmitted light regardless of which material it is coming from.



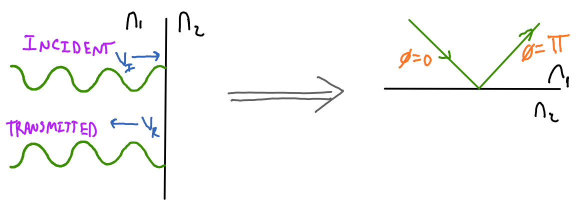
Reflected light: ($n_1 > n_2$)

- No phase shift for light reflected off a lower index of refraction.



Reflected light: ($n_1 < n_2$)

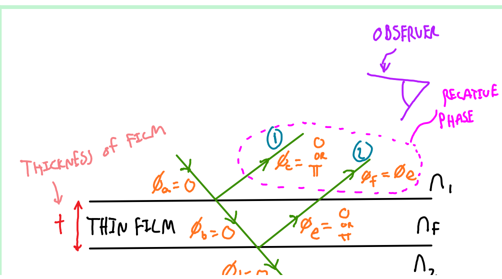
- A π phase shift (i.e. 180° phase shift) for light reflected off a higher index of refraction.



*Note: In the above three rules, the incident light is perpendicular to the boundary surface. When looking at the images on the right hand side of the arrow, the wave is represented with an arrow at an angle only for clarity. Basically, if the rays representing the wave were drawn perpendicular to the surface it would be confusing because the reflected rays travel back the same path. For this class, we will only consider light that is traveling perpendicular to the boundary surface (e.g. normal incidence), however we will use diagrams that look as if the light is at an angle but only to help identify which wave is transmitted and reflected.

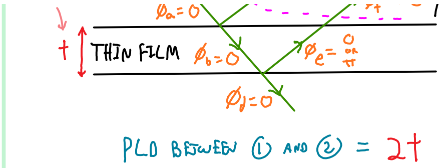
Mathematical model

We can now combine our rules for phase shifts across boundaries to formulate a mathematical model as shown below. It is highly recommended you draw the diagram below for every thin film problem to help identify the phase shifts.



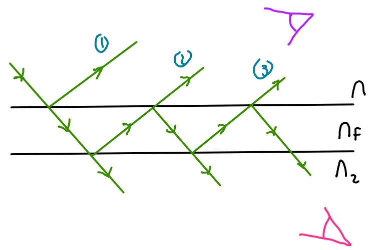
$m = 0, 1, 2, 3, 4, 5 \ldots$

RELATIVE PHASE	CONSTRUCTIVE	DESTRUCTIVE
EVEN (IN PHASE) $\phi_c = \phi_f$	$2t = m\lambda_F$	$2t = (m+\frac{1}{2})\lambda_F$
ODD (180° OUT OF		



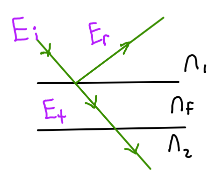
EVEN (IN PHASE) $\phi_c = \phi_f$	$2t = m\lambda_F$	$2t = (m+\frac{1}{2})\lambda_F$
ODD (180° OUT OF PHASE) $\phi_c \neq \phi_f$	$2t = (m+\frac{1}{2})\lambda_F$	$2t = m\lambda_F$

You might be wondering why only two reflections are shown above: one reflection from the top of the film and the other form the bottom of the film. You are right to question this diagram. In general, the reflected ray from the bottom reflects again at the top surface and so on. This process looks like the figure below.



Multiple reflections and transmissions occur as seen above. The net effect does not change our mathematical model, eave adjacent wave (1 and 2, 2 and 3, 3 and 4…) maintain the same phase relationship as the first set (1 and 2) and the path length difference between each adjacent wave is still 2t (you should try confirming these statements). Showing more reflections does introduce an interesting idea; you can be on either side of the thin film to observe interference.

Now is a good time to remind ourselves about the energy transported via light waves. When the initial incident light wave reaches the boundary it carries with it energy E_i. At the boundary, the initial wave gets partially reflected and partially transmitted. Thus some of the initial energy E_i gets sent along with the transmitted E_t and some of it goes with the reflected wave E_r. If there is no loss of energy to the medium, then energy is conserved ($E_i = E_t + E_r$). Below is a physical representation of this process.



$$E_i = E_t + E_r$$

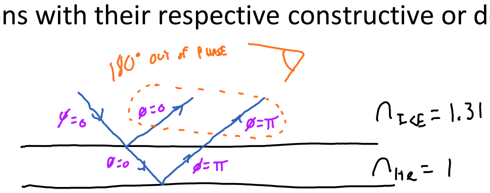
Recall from our initial energy discuss in lecture 25 that the energy in a traveling wave is proportional to both the frequency and amplitude squared.

$$E \propto f^2 A^2$$

Keep in mind the frequency is a constant for the initial, transmitted and reflected wave. Thus the amplitude of the transmitted and reflected wave must change (i.e. the amplitude of the electric and magnetic fields for the transmitted and reflected waves are smaller than the initial). Since energy is proportional to intensity, this means that the intensity is also decreased for the transmitted and reflected waves.

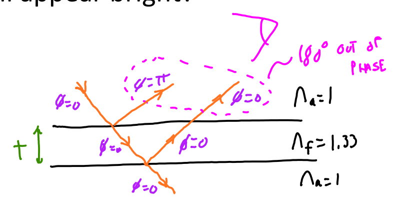
PRACTICE: Light traveling through ice (n=1.31) is incident on a thin film (thickness t) of Helium gas (n=1) that has diamond (n=2.41) on the other side. Match the following conditions with their respective constructive or destructive interference.

(a) $2t = (m + 0.5)\lambda$ → 1. Constructive
(b) $2t = m\lambda$ → 2. Destructive



$n_f = 2.41$

PRACTICE: Orange light (λ_{vacuum} = 611 nm) shines on a soap film with an index of refraction of 1.33. On either side of the soap film is air. What are the minimum thicknesses of the film which the orange light will appear bright?



PLD = CONST. 180° OUT OF PHASE

$$2t = (m+\tfrac{1}{2})\lambda_f$$

$$2t = (m+\tfrac{1}{2})\frac{\lambda_v}{n_f}$$

$$n_v\lambda_v = n_f\lambda_f$$

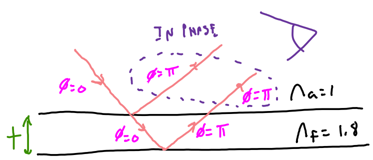
$$\lambda_f = \frac{n_v}{n_f}\lambda_v$$

$$\lambda_f = \frac{\lambda_v}{n_f}$$

m	t
0	115 nm
1	345 nm
2	575 nm
⋮	⋮

PRACTICE: The Decepticons are building a new top secret skin for their jets that makes them invisible to the Transformer's X-Band radar detectors. The X-Band operates at 12 GHz and the material they want to make the skin out of has an index of refraction of 1.80. What is the minimum thickness of the film?

(a) 32 nm
(b) 15 nm
(c) 11 cm
(d) 4.2 cm
(e) 783 mm
(f) 0.35 cm



$n_{METAL} > n_f$

PLD = DEST. IN PHASE

$$2t = (m+\tfrac{1}{2})\lambda_f$$

$$2t = (m+\tfrac{1}{2})\frac{\lambda_a}{n_f}$$

$$n_a\lambda_a = n_f\lambda_f$$

$$\lambda_f = \frac{\cancel{n_a}^{1}}{n_f}\lambda_a$$

$$\lambda_f = \frac{\lambda_a}{n_f}$$

m	t
0	0.35 cm
1	
2	

QUESTIONS FOR DISCUSSION:

(1) Can you think of any applications that would benefit from the interference effects from thin films?