ROTATIONAL MECHANICS

ANGLE "POSITION" [UNITLESS] $\equiv \theta$ ← MEASURED IN RADIANS

ANGULAR VELOCITY $\frac{1}{[T]} \equiv \omega$

ANGULAR ACCELERATION $\frac{1}{[T]^2} \equiv \alpha$

AVERAGE ANGULAR VELOCITY $\frac{1}{[T]} \equiv \bar{\omega} = \frac{\Delta\theta}{\Delta t}$

AVERAGE ANGULAR ACCELERATION $\frac{1}{[T]^2} \equiv \bar{\alpha} = \frac{\Delta\omega}{\Delta t}$

ARCH LENGTH $[L] \equiv S = \Delta\theta \cdot r$

TANGENTIAL COMPONENT OF VELOCITY $\frac{[L]}{[T]} \equiv V_t = \omega r$

TANGENTIAL COMPONENT OF ACCELERATION $\frac{[L]}{[T]^2} \equiv a_t = \alpha r$

RADIAL COMPONENT OF ACCELERATION $\frac{[L]}{[T]^2} \equiv a_r = \frac{V_t^2}{r} = \omega^2 r$

PERIOD $[T] \equiv T$

FREQUENCY $\frac{1}{[T]} \equiv f = \frac{1}{T}$

VELOCITY $\frac{[L]}{[T]} \equiv \vec{V} = \langle \overset{0}{V_r}, V_t, V_z \rangle = \langle 0, \omega r, V_z \rangle$

ACCELERATION $\frac{[L]}{[T]^2} \equiv \vec{a} = \langle a_r, a_t, \overset{0}{a_z} \rangle = \langle \omega^2 r, \alpha r, 0 \rangle$

OR

$= \langle \frac{V_t^2}{r}, \alpha r, 0 \rangle$

KINEMATIC EQUATIONS

*CONSTANT α

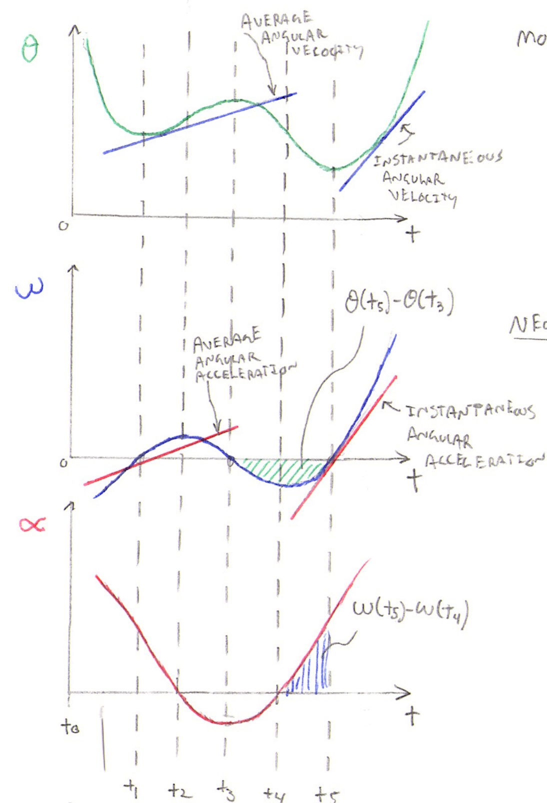
$$\theta_f = \theta_i + \omega_i \Delta t + \tfrac{1}{2}\alpha \Delta t^2$$

$$\omega_f = \omega_i + \alpha \Delta t$$

$$\omega_f^2 = \omega_i^2 + 2\alpha\Delta\theta$$

* CONVENTION: $\begin{cases} CCW\ (+) \\ CW\ (-) \end{cases}$

$$\underset{\text{AREA}}{\overset{\text{SLOPE}}{\Delta\theta(t) \rightleftarrows \omega(t)}} \qquad \underset{\text{AREA}}{\overset{\text{SLOPE}}{\omega(t) \rightleftarrows \alpha(t)}}$$



STABILITY - OBJECT OR SET OF OBJECTS UNDER GRAVITY IS STABLE IF THEIR TOTAL COM IS INSIDE FURTHEST MOST NORMAL FORCE.

CENTER OF MASS (COM) $[L] \equiv X^{tot}_{com} = \frac{\sum X_i m_i}{\sum m_i}$

$$X^{tot}_{com} = \frac{X_{1com} m_1 + X_{2com} m_2 + \dots}{m_1 + m_2 + \dots}$$

*SAME FOR Y^{tot}_{com}

TORQUE $\frac{[M][L]^2}{[T]^2} \equiv \vec{\tau} = \vec{r} \times \vec{F}$ ← CROSS PRODUCT

* CONVENTION $\begin{cases} CCW(+) \\ CW(+) \end{cases}$

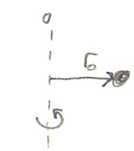
$|\vec{r} \times \vec{F}| = |\vec{r}||\vec{F}| \sin\theta$ ← SMALLEST ANGLE BETWEEN THE TWO VECTORS

$|\vec{r} \times \vec{F}| = r_\perp |\vec{F}| = |\vec{r}| F_\perp$

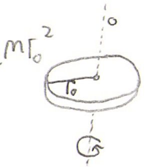
MOMENT OF INERTIA $[M][L]^2 \equiv I_o = \sum_i m_i r_{oi}^2$

AXIS "O"

POINT PARTICLE $I_o = M r_o^2$



DISK $I_o = \frac{1}{2} M r_o^2$



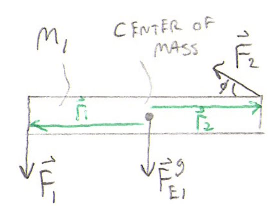
NEWTON'S 2ND LAW (RIGID BODIES)

$$\sum \vec{\tau}_{EXT,o} = I_o \vec{\alpha}_o \rightarrow \sum \bar{\tau}_{EXT,o} = I_o \frac{\Delta\omega_o}{\Delta t}$$

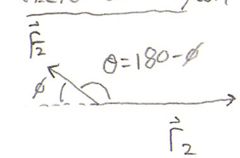
AXIS "O"

e-FBD

EXAMPLE: •UNIFORM DENSITY



VECTOR-OF $\tau_{2,com}$



$\tau_{2,com} = +|\vec{r}_2||\vec{F}_2| \sin(180-\phi)$

+ FROM CCW

ANGULAR MOMENTUM $\frac{[M][L]^2}{[T]} \equiv \vec{L} = \vec{r} \times m\vec{V}$ $\begin{cases} |\vec{L}| = m|\vec{r}||\vec{V}|\sin\theta \\ |\vec{L}| = m r_\perp |\vec{V}| \\ |\vec{L}| = m|\vec{r}| V_\perp \end{cases}$

OR

$\vec{L} = I\vec{\omega}$

ANGULAR IMPULSE $\frac{[M][L]^2}{[T]} = \sum \bar{\tau}_{EXT,o} \Delta t = \Delta L \sim \Delta L = I_o \Delta\omega_o$

.. IF... $\sum \bar{\tau}_{EXT,o} \Delta t = 0$ ....

$\Delta L = 0$ ~ CONSERVATION OF ANGULAR MOMENTUM

ROTATIONAL KINETIC ENERGY $\frac{[M][L]^2}{[T]^2} \equiv KE_R = \frac{1}{2} I\omega^2 = \frac{1}{2} L\omega$